Implicitization of Geometric Objects under Affine Transformations using Gröbner Walks

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Extended Abstract

In computer aided geometric design, a geometric object is often a combination of several patches. These patches may actually come from a smaller subset of source patches using affine transformations such as rotation, scale and shear. As the result, some algebraic or even transcendental constants such as $\sin(\pi/3)$ and $\cos(\pi/5)$ may appear on the equations, which makes the implicitization much harder.
Usually, parametric forms such as Bezier patches or NURBS are used for designing the objects. But implicit forms are also needed in several occasion such as for finding the intersection of the patches or for finding whether or not a point is on a patch. There are several methods for implicitization such as resultants, Gröbner bases and moving curves/surfaces.
The method of Gröbner bases has the important advantage that they can solve the implicitization problem in full generality. However, Gröbner bases are known to be very slow in implicitizing bicubic patches.
This paper investigate the use of new theoretical results in the method of Gröbner bases over the last five years to improve the efficiency of algorithms for implicitization.
The main result of this paper is that we can dramatically improve the efficiency of implicitization algorithms using the deterministic Gröbner walk conversion. The main ideas are that: first, since the parametric equations of a patch is already or very close to a Gröbner basis with respect to any elimination order for the parameters, one can convert a Gröbner basis by partitioning the computation of the basis into several smaller computations following a path in the Gröbner fan of the ideal generated by the system of equations. The method works with ideals of zero-dimension as well as positive dimension. Typically, the target point of the walking path lies on

the intersection of very many cones, which ends up with initial forms of a considerable number of terms and therefore huge intermediate polynomial systems. The deterministic method in [Tra00] to vary the target point can ensure the generality of the position, i.e. we always have just a few terms in the initial forms. Second, if a patch is an image of a known patch under an affine transformation, then one can use the structure of the Gröbner bases to convert from the basis of the known patch rather than computing the Gröbner bases from scratch. This approach, initiated in [Hon97] will speed up the computation, especially when the image patch has algebraic or transcendental constants.
Our experiments show the superlative performance of our improved Gröbner walk method in comparison with the traditional ones. The average performance is 5×10^2 to 10^3 times faster than the direct computation of the reduced Gröbner basis with respect to pure lexicographic term order (using the Buchberger algorithm and the sugar cube strategy).

References

[Hon97] Hoon Hong. Groebner basis under composition I. *Journal of Symbolic Computation*, 11, 1997.

[Tra00] Quoc-Nam Tran. A fast algorithm for Gröbner basis conversion and its applications. *Journal of Symbolic Computation*, 30:451–468, 2000.