ACCLAIM
Periodic Progress Report
of
RISC

Hoon Hong
Andreas Neubacher

June 19, 1995

1 Clarification

As we already mentioned in the last PPR (1994), RISC is in Austria, an EFTA country at the time when ACCLAIM was proposed. Thus, it is not funded by ESPRIT, rather by the Austrian Science Foundation. The funding started from July of 1993 because

- The Austrian Science Foundation carried out its own refereeing process, approving the project on March of 1993.
- There was some administrative delay.

Thus, RISC was not able to carry out the work as specified in the technical annex. Upon the approval of EC representative (Dr. Rouhana), RISC shifted the work plan by 1 year. Thus, in this annual report, we report the work that was originally planned for 1994.

2 Management Report (9/1994 – 2/1994)

Workpart Code:	Task 2.9
Status (Start/Continue/Finished) this period:	Finished
Status (Start/Continue/Finished) next period:	-
Human resources in person-months (PM) this period:	6
Human resources in person-months (PM) next period:	-
Organization:	RISC

3 Update on the Worldwide State-of-the-Art

4 Partner's Reports

4.1 RISC

Achievements

The main achievements of RISC during last year were to investigate various aspects of non-linear real and complex algebraic constraints solving and to develop application software.

We concentrated on the following research direction:

- **Satisfiability of constraints**: The problem is to decide if a given set of algebraic constraints is satisfiable.

  In [3] we proposed an approach for dramatically reducing computation time for the Cylindrical Algebraic Decomposition method in "real-world" applications.

  In [5] we describe the implementation of a Quadratic Form method for counting the number of solutions of a set of algebraic constraints.

- **Approximate solutions of constraints**: The problem is to approximate the real and complex solutions of a given set of algebraic constraints.

In [2] we have developed a method which combines non-linear interval arithmetic and linear methods the narrowing idea and the fixed point idea.

In [8, 7] we describe interval methods for algebraic constraints.

- **Exact solutions of constraints**: The problem is to exactly solve systems of constraints over the real and complex domain.

  In [4] we describe a method for triangularizing systems of algebraic constraints by the Involution method for computing Grbner bases.

  In [1] we describe a method for finding maximal solutions of constraints of the form $f(x) > 0$.

- **Software**: We implemented several methods for dealing with non-linear constraints as listed above. These implementations are collected in the *Sturm library*, a library of C++ functions for constraint solving.

  Based upon this we developed a CLP system RISC-CLP(Tree(Δ)) [6] which combines some of these implementations with LP.

Presentations

- Hoon Hong: *Generic Quantifier Elimination*

  IMACS symposium on the application of computer algebra, Albuquerque, New Mexico, USA, 1995

- Hoon Hong: *Approximate Quantified Constraint Solving*

  ACCLAIM Workshop, Pisa, 1995

5 Task Reports

5.1 Work package 2b: Numerical Constraints

Task 2.9: Algebraic Constraints

Partner Responsible. RISC

Other Partners. UML, SICS

The main goal of this task is to develop algorithms and software for solving non-linear algebraic constraints over real and complex numbers. Research on constraints over reals is essentially finished. Work on constraints over complexes is based on the work done in the real domain, since the approach taken for dealing with complex constraints is to represent complex constraints by systems of real constraints.

- **Satisfiability of constraints**: The problem is to decide if a given set of algebraic constraints is satisfiable.

  In [3] we proposed an approach for dramatically reducing computation time for the Cylindrical Algebraic Decomposition method in "real-world" applications.

  In [5] we describe the implementation of a Quadratic Form method for counting the number of solutions of a set of algebraic constraints.

- **Approximate solutions of constraints**: The problem is to approximate the real and complex solutions of a given set of algebraic constraints.

  In [2] we have developed a method which combines non-linear interval arithmetic and linear methods the narrowing idea and the fixed point idea.

  In [8, 7] we describe interval methods for algebraic constraints.

- **Exact solutions of constraints**: The problem is to exactly solve systems of constraints over the real and complex domain.

  In [4] we describe a method for triangularizing systems of algebraic constraints by the Involution method for computing Grbner bases.

  In [1] we describe a method for finding maximal solutions of constraints of the form $f(x) > 0$.

In order to test these algorithms/techniques, we implemented them in C++ and have also developed a prototype CLP system "RISC-CLP(Tree(Δ))" [6] that uses them. As the methods stabilities, we plan to incorporate them into other more professional CLP systems such as PROLOG IV, etc.

Deliverables due 2Q94 (delayed). D2.9/3, D2.9/4

D2.9/3 Reports on work for an incremental non-linear constraint solver for complexes.

D2.9/4 A prototype system for an incremental non-linear constraint solver for complexes.

Deliverables due 2Q95. D2.9/5, D2.9/6 – These deliverables are delayed due to the one-year delay in starting project work described in Section 1.

6 Deliverables

D2.9/1: Reports on Non-linear constraint solver for reals

Partner Responsible. RISC

Other Partners. UML, SICS

This deliverable consists of 9 attached papers reporting the work done on various aspects of non-linear constraint solving.

- Satisfiability of constraints: Attachment 1

- Approximate solutions of constraints: Attachments 2, 3
- Exact solutions of constraints: Attachment 4, 5
- CLP applications: Attachment 6

Attachment 1.

Title. Approximate Quantified Constraint Solving **Authors.** Hoon Hong and Wolfgang Stöcher

When solving equational constraints it is often required to quickly check whether there exist solutions at all and to compute the number of solutions.

In this report we describe our experiences with implementing an algorithm for counting the common real roots of a given polynomial system. Our approach is based on multivariate Sturm Theory (based on Pederson's results) and the Involution method for computing Gröbner bases.

Attachment 2.

Title. Interval Methods for Bounding the Range of Univariate Polynomials

Authors. Volker Stahl

The overestimation of the range of functions is a fundamental problem which arises in many interval methods for constraint solving. We will not treat the problem in its full generality, rather we restrict ourselves to polynomials. Apart from linear functions, for which range computation is trivial, polynomials are the "simplest" subclass of elementary transcendental functions. We can therefore expect more efficient methods and more refined results for polynomials than for arbitrary functions. The practical importance of polynomials is out of question, therefore it seems worthwhile to study them separately. In this paper we consider univariate polynomials, the multivariate is treated in another paper.

Attachment 3.

Title. Interval Methods for Bounding the Range of Multivariate Polynomials

Authors. Volker Stahl

This is the companion paper to Attachment 2. Here, results for multivariate polynomials are described.

Attachment 4.

Title. Implementation of an Involutive Method for Computing Gröbner Bases of Zero-dimensional Ideals

Authors. Hoon Hong and Andreas Neubacher

A fundamental problem in constraint solving is that of solving systems of non-linear equations. The method of Gröbner basis generalizes the well known Gaussian elimination of the linear case to the nonlinear case. In this paper, we give an overview of the mathematical foundations of a new algorithm by A.Yu. Zharkov and Yu.A. Blinkov for computing Gröbner bases for zero-dimensional ideals. Furthermore, a report on an implementation of this algorithm with descriptions of experimental results (esp. with regard to runtime behavior) is given.

Attachment 5.

Title. On the Search of the Maximum of a Polynomial

Authors. A.Y. Uteshev, T.M. Cherkasov

For a real polynomial $f(X)$ of k variables the problem of finding $\max_{X \in \mathbf{R^k}} f(X)$ is investigated by reducing it to that of searching for the real roots of a polynomial in one variable: $F(z) := \prod_{j=1}^{N}(f(X_j) - z)$, where X_j is a critical point of $f(X)$. The use of Hermite method of separation of real solutions of algebraic equation systems permits one to construct together with $F(z)$ also its Sturm series, and restore the coordinates of corresponding critical point. The problem of finding the $\max f$ in the set defined by the real polynomial inequality $G(X) \geq 0$ is also discussed.

Attachment 6.

Title. RISC-CLP(Tree(Δ)) A Constraint Logic Programming System with Parametric Domain

Authors. Hoon Hong and Stefan Ratschan

Current implementations of constraint logic programming languages (like CLP($\Re$), CHIP or RISC-CLP(Real) support constraint solving over a certain fixed domain. In this paper a system is presented which gives the possibility to instantiate a constraint logic programming language with an arbitrary constraint domain. The interface between the system and such a constraint

domain is given and the extensions to the Warren Abstract Machine (WAM) for this implementation are presented.

Further Use.

Attachments 1 – 5 describe algorithms and methods for constraint solving. The Attachment 6 describes a prototype application (a CLP system).

Some of the algorithms in attachments 1 – 5 are used in the implementation of the prototype system. It is planned to use all the algorithms in a final version of the system described in the Attachment 6.

Further work is required for integrating these methods into professional CLP systems such as PROLOG IV, Oz, AKL developed by other partners of ACCLAIM.

Impact.

Currently the main impacts of these deliverable are

- publication, the dissemination of scientific ideas to public (Attachments 5 and 6);
- implementation, gaining insight into the runtime behavior and real-world applicability of the new methods.

D2.9/4: An incremental non-linear constraint solver for complexes

Partner Responsible. RISC

Other Partners. UML, SICS

Attachment 1.

Title. RISC-CLP(Tree(Δ))

Authors. Stefan Ratschan, Hoon Hong

This package is a prototype CLP system for testing some of the methods developed in the frame of Deliverable D2.9/3.

The system implements an extension to the Warren Abstract Machine (WAM) which allows instantiation of a constraint logic programming language with

an arbitrary constraint domain. It is written in C++, with the constraint part relying upon the *Sturm* library, a C++ library comprising most of the constraint solving methods developed for ACCLAIM.

References

[1] T.M. Cherkasov A.Y. Uteshev. On the search of the maximum of a polynomial. Technical Report 95-16, Research Institute for Symbolic Computation, Johannes Kepler University A-4040 Linz, Austria, 1995.

[2] H. Hong. Approximate quantified constraint solving. In *ACCLAIM Workshop, Pisa*, 1995.

[3] H. Hong. Generic quantifier elimination. In *IMACS symposium on the application of computer algebra, Albuquerque, New Mexico, USA*, 1995.

[4] H. Hong and A. Neubacher. Implementation of an involutive method for computing gröbner bases of zero-dimensional ideals. Technical report, Research Institute for Symbolic Computation, Johannes Kepler University A-4040 Linz, Austria, 1995.

[5] H. Hong and W. Stöcher. The quadratic form method for counting real zeros. Technical report, Research Institute for Symbolic Computation, Johannes Kepler University A-4040 Linz, Austria, 1995.

[6] Stefan Ratschan. RISC-CLP(Tree(Δ)) A Constraint Logic Programming System With Parametric Domain. Technical Report 94-76, RISC-Linz, Johannes Kepler University, Linz, Austria, 1994.

[7] V. Stahl. Interval methods for bounding the range of multivariate polynomials. Technical report, Research Institute for Symbolic Computation, Johannes Kepler University A-4040 Linz, Austria, 1995.

[8] V. Stahl. Interval methods for bounding the range of univariate polynomials. Technical report, Research Institute for Symbolic Computation, Johannes Kepler University A-4040 Linz, Austria, 1995.