Combining Level-Saturation Strategies and Meta-Variables for Predicate Logic Proving in *Theorema*

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The problem of generating proofs that can be easily read by non-specialists has raised an increased interest in the recent years. One of the main advantages of "natural style" proving is that one can implement as proof procedures the techniques used by mathematicians, and one can also use advanced solvers for particular domains, in the form of special inference rules. This approach is followed by the *Theorema* system, which aims at integrating proving, computing, and solving in one coherent system based on *Mathematica* software (see [2, 1] and `http://www.theorema.org`).

The need for readability of the result, but even more the flexibility needed for dynamic extension of the prover with special inference rules, as well as for dynamic interaction with special "black box" computing and solving engines makes it difficult to use established efficient techniques such as resolution, tableaux, or connection methods. Therefore, the usual approach is to employ a sequent calculus (like Gentzen or similar)—whose main draw-back is the lack of determinism in selecting the right replacements in some of the quantified rules (elimination of $\forall$ in assumptions and of $\exists$ in goal). The corresponding problem in "natural style" deduction is the finding of "witnessing terms" (especially for existential goals).

We approach the problem of finding witnessing terms in proofs by the method of meta-variables. The method is based on a special technique for finding meta-substitutions when the proof search is performed in an `AND-OR` deduction tree (see also [3]).

The implementation of the method has the following distinctive characteristics:

- The method can be used together with any set of inference rules, in particular with any special provers.
- The method can be used together with any special (constraint) solvers and rewrite engines based on implicit or explicit equality sets.
- The proof is obtained as soon as possible, and inconsistent meta-substitutions are eliminated as soon as they can be detected.
- Proof sub-trees are not destroyed, but they are reused as much as possible.
- The method can be used together with any strategy for constructing the proof tree.

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However, our experiments showed that an unrestricted introduction of meta-variables leads to large deduction trees. Therefore, in addition to the standard rules of the sequent calculus, we are introducing more special rules inspired by the natural style of proving by the working mathematician (e.g. instantiation of the goal if it matches some ground assumption), which are applied before the introduction of the meta-variables. Such rules make proof search faster; the proofs themselves are shorter and much more human-like.

In particular, we experiment the meta-variables method in conjunction with the level-saturation strategy which is currently implemented in the predicate-logic prover of *Theorema*. This strategy is used both in forward (from assumptions to the goal) and in backward mode (from goal to the assumptions). The direction can be chosen by the user by setting certain options, the mixed mode (alternating directions) is also available. For proofs in elementary calculus (e. g. limit of sum) we detected that the backward strategy coupled with meta-variables can generate quite natural proofs, and it may be even used for reducing the proof problem to simpler proof problems (when some of the needed assumptions are not given in advance).

References

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