The Facilities of Theorema for Teaching Logic and Mathematics*

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Abstract

The TH∃OREM∀ project aims at supporting, within one consistent logic and one coherent software system, the entire mathematical exploration cycle consisting of

- introducing new notions (functions, predicates, algorithms) by definitions or axioms
- conjecturing and proving / disproving mathematical facts about the new notions w.r.t. knowledge bases consisting of definitions, axioms, lemmata, theorems etc.
- extracting methods and algorithms from proved facts
- applying algorithms to input data.

While one long term goal of the system is to be used by the working mathematician in writing scientific papers, textbooks, exploring new theories, one imediate use of the software is (and will also be in the future) the on-line assistance for teaching logics and mathematics [first] at elementary and at undergraduate level. (In fact, our experience shows that the system is also useful for learning proving techniques at graduate and postgraduate levels.) Educational

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experiences with an earlier version of TH∃OREM∀ are reported in [1, 7]. In the present paper we report on the current facilities of the system which are useful for educational purposes.

The TH∃OREM∀ system is implemented using the computer algebra system *Mathematica* and it is based on earlier work of Buchberger [2, 3]. The first version of the system is described in [6], and the present version is presented in [5]. Additionally, the current version of the system and many proof examples are available on the home page of the project: `www.theorema.org`.

The objectives and features that distinguish the TH∃OREM∀ project from the other, similar, projects are the following:

- We emphasize "theory exploration" over "theorem proving", i.e. the goal of TH∃OREM∀ is to support the user in formally developing and proving big portions of coherent mathematical texts (like lecture notes and textbooks) rather than in proving isolated theorems, see [4].
- We consider both computer-support in the "routine" reductions of proof problems to known black-box techniques and also the computer-supported invention of "ingenious" proofs by strong special techniques (e.g. algebraic techniques) based on nontrivial mathematics.
- We emphasize automated proof generation over automated proof checking.
- We emphasize, in automated routine proofs, the importance of generating human-readable proof presentations in a "natural style".
- We integrate proving, solving, and simplifying, as the main three formal activities in exploring mathematics, in one coherent system.
- We integrate many different general and special provers, solvers, and simplifiers into one system, i.e. TH∃OREM∀ is a multi-method system.
- We use a commercial mathematical software system (*Mathematica* [8]) as our programming environment so that TH∃OREM∀ is available on all platforms.
- We emphasize the importance of convenient, two-dimensional, extensible syntax and graphical illustration.

The main features of TH∃OREM∀ are:

- a TH∃OREM∀ formal text language for defining new notions, algorithms and their properties, for building up mathematical theories and using them;
- a new predicate logic prover that integrates proving, simplifying and solving steps in a systematic way: the PCS prover (this prover combines techniques for predicate-logic proving, rewriting using universal formulae, and solving in specific domains using black-box engines),

- a special prover for set theory, integrated with the PCS prover,
- a general propositional and predicate-logic proving, using forward and backward level-saturation strategies, and meta-variables;
- provers by induction and by simplification over integers and lists,
- tools for proof simplification by post-processing – removing superfluous branches and steps,
- integration of powerful special provers (the Groebner Bases prover and the Gosper-Zeilberger prover) as well as of other "external" predicate logic provers (e. g. by resolution),
- tools for extending existing provers by general strategies: the cascade strategy and its application to induction,
- tools for improving the presentation of the formal text by new "logicographic" symbols, which represent in an intuitive way the user-defined predicates and functions.

The various features of the system will be demonstrated interactively during the presentation.

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