Exploring the Trapezoid

- Textbook formulation:

Let *ABCD* be a trapezoid ($AB \parallel CD$); *M*, *N* the midpoints of *AD* resp. *BC*; *P* the intersection of the two diagonals; *X* the intersection of line *MN* with the diagonal *BD*; *Q*, *S* the intersections of the line passing through *P* and parallel to *AB* with lines *BC* resp. *AD*. Explore the geometric configuration that results from the previous constructions.

- Input to the system:

```
Assumptions["Trapezoid1", any[A, B, C, D],
 triangle[A, B, C] ∧
  pon[D, pline[C, A, B]] ∧
  line[B, C] ∧
  line[A, D]]
```

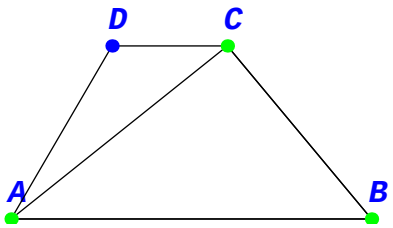
- Input to the system:

```
KnowledgeBase["Trapezoid_g", any[A, B, C, D],
 {{A, {0, 0}}, {B, {250, 0}}, {C, {150, 120}}, {D, {70, 120}}}]
```

- Input to the system:

```
Simplify[Assumptions["Trapezoid1"],
 by -> GraphicSimplifier, using -> KnowledgeBase["Trapezoid_g"]]
```

- Output generated automatically by the system



- End of output

- Input to the system:

```
kb1 = Explore[Assumptions["Trapezoid1"], by → AreaExplore,
   ExplorerOptions → {GraphicPoints → KnowledgeBase["Trapezoid_g"]}];
```

- Inserted notebook generated automatically by the system

Select construction, then press the button to view the graph

```
PrintGraph[■]
```

For any points {*A*, *B*, *C*, *D*} that satisfy the following constructions:

```
triangle[A, B, C] ∧                                    ,
 pon[D, pline[C, A, B]] ∧ line[B, C] ∧ line[A, D]
```

recommended position of points for drawing,

```
{{A, {0, 0}}, {B, {250, 0}},
 {C, {150, 120}}, {D, {70, 120}}}
```

we can prove, that:

There are no incidental points.

There are no more than two collinear points.

Parallel lines:

$AB \parallel CD$

There are no perpendicular lines.

There are no segments with proportional length

Triangles with proportional area:

Area[ABC] ≅ Area[ABD]

Area[ACD] ≅ Area[BCD]

- End of inserted notebook

- Input to the system:

```
Assumptions["Trapezoid2", any[A, B, C, D, M, N],
 midpoint[M, A, D] ∧
  midpoint[N, B, C] ∧
  line[M, N]]
```

- Input to the system:

```
kb2 = Explore[Assumptions["Trapezoid2"],
   by → AreaExplore, using → kb1, producing -> Tr2];
```

- Inserted notebook generated automatically by the system

Select construction, then press the button to view the graph

```
PrintGraph[■]
```

For any points {A, B, C, D, M, N} that satisfy the following constructions:

```
triangle[A, B, C] ∧ pon[D, pline[C, A, B]] ∧
 line[B, C] ∧ line[A, D] ∧ midpoint[M, A, D] ∧
 midpoint[N, B, C] ∧ line[M, N]
```

we can prove, that:

There are no incidental points.

Collinear points: {{A, D, M}, {B, C, N}}

Parallel lines:

$AB \parallel CD \parallel MN$

There are no perpendicular lines.

Segments with proportional length:

$|AD| \cong 2|AM| \cong 2|DM|$

$|BC| \cong 2|BN| \cong 2|CN|$

Triangles with proportional area:

- End of inserted notebook

- Input to the system:

```
Assumptions["Trapezoid3", any[A, B, C, D, M, N, X, P, Q, S],
 inter[X, line[M, N], line[B, D]] ∧
  inter[P, line[A, C], line[B, D]] ∧
  inter[Q, line[B, C], pline[P, A, B]] ∧
  inter[S, line[P, Q], line[A, D]] ∧
  line[M, C] ∧
  line[A, Q]]
```

- Input to the system:

```
kb3 = Explore[Assumptions["Trapezoid3"],
   by → AreaExplore, using → kb2, producing → Tr3,
   ExplorerOptions → {GraphicPoints → KnowledgeBase["Trapezoid_g"]}];
```

- Inserted notebook generated automatically by the system

Select construction, then press the button to view the graph

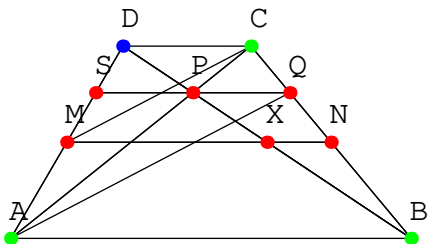
```
PrintGraph[■]
```

For any points {**A**, **B**, **C**, **D**, **M**, **N**, **X**, **P**, **Q**, **S**} that satisfy the following constructions:

```
triangle[A, B, C] ∧ pon[D, pline[C, A, B]] ∧ line[B, C] ∧
 line[A, D] ∧ midpoint[M, A, D] ∧ midpoint[N, B, C] ∧
 line[M, N] ∧ inter[X, line[M, N], line[B, D]] ∧
 inter[P, line[A, C], line[B, D]] ∧
 inter[Q, line[B, C], pline[P, A, B]] ∧
 inter[S, line[P, Q], line[A, D]] ∧ line[M, C] ∧ line[A, Q]
```

recommended position of points for drawing,

```
{{A, {0, 0}}, {B, {250, 0}},
 {C, {150, 120}}, {D, {70, 120}}}
```



we can prove, that:

There are no incidental points.

Collinear points: {{**A**, **D**, **M**, **S**}, {**M**, **N**, **X**}, {**B**, **C**, **N**, **Q**}, {**B**, **D**, **X**, **P**}, {**A**, **C**, **P**}, {**P**, **Q**, **S**}}

Parallel lines:

AB ∥ **CD** ∥ **MN** ∥ **PQ**

CM ∥ **AQ**

BM ∥ **DQ**

AN ∥ **CS**

$DN \parallel BS$

There are no perpendicular lines.

Segments with proportional length:

$|AD| \cong 2|AM| \cong 2|DM|$

$|BC| \cong 2|BN| \cong 2|CN|$

$|BD| \cong 2|BX| \cong 2|DX|$

$|AB| \cong 2|MX|$

$|CD| \cong 2|NX|$

$|PQ| \cong |PS| \cong \frac{1}{2}|QS|$

Triangles with proportional area:

$\text{Area}[ABC] \cong \text{Area}[ABD] \cong 2\text{Area}[ABM] \cong 2\text{Area}[BDM] \cong 2\text{Area}[ACN] \cong 2\text{Area}[ABN] \cong 2\text{Area}[ADX] \cong 2\text{Area}[ABX] \cong 2\text{Area}[BMQ] \cong 2\text{Area}[ANS] \cong 4\text{Area}[AMX] \cong 4\text{Area}[BMX] \cong 4\text{Area}[CMX] \cong 4\text{Area}[DMX]$

$\text{Area}[ACD] \cong \text{Area}[BCD] \cong 2\text{Area}[ACM] \cong 2\text{Area}[CDM] \cong 2\text{Area}[BDN] \cong 2\text{Area}[CDN] \cong 2\text{Area}[BCX] \cong 2\text{Area}[CDX] \cong 2\text{Area}[CMQ] \cong 2\text{Area}[DNS] \cong 4\text{Area}[ANX] \cong 4\text{Area}[BNX] \cong 4\text{Area}[CNX] \cong 4\text{Area}[DNX]$

$\text{Area}[BCM] \cong \text{Area}[ADN] \cong 2\text{Area}[AMN] \cong 2\text{Area}[BMN] \cong 2\text{Area}[CMN] \cong 2\text{Area}[DMN]$

$\text{Area}[ADP] \cong \text{Area}[BCP] \cong \text{Area}[AMQ] \cong \text{Area}[ACQ] \cong \text{Area}[BDQ] \cong \text{Area}[DMQ] \cong \text{Area}[ACS] \cong \text{Area}[BNS] \cong \text{Area}[BDS] \cong \text{Area}[CNS] \cong \frac{1}{2}\text{Area}[ADQ] \cong \frac{1}{2}\text{Area}[BCS] \cong 2\text{Area}[AMP] \cong 2\text{Area}[BNP] \cong 2\text{Area}[CNP] \cong 2\text{Area}[DMP] \cong 2\text{Area}[BXQ] \cong 2\text{Area}[DXQ] \cong 2\text{Area}[BXS] \cong 2\text{Area}[DXS]$

$\text{Area}[ABP] \cong \text{Area}[ABQ] \cong \text{Area}[ABS] \cong 2\text{Area}[ANP] \cong 2\text{Area}[BMP] \cong 2\text{Area}[AXS]$

$\text{Area}[CDP] \cong \text{Area}[CDQ] \cong \text{Area}[CDS] \cong 2\text{Area}[CMP] \cong 2\text{Area}[DNP] \cong 2\text{Area}[CXQ]$

$\text{Area}[ACX] \cong 2\text{Area}[MNP] \cong 2\text{Area}[MNQ] \cong 2\text{Area}[MNS]$

$\text{Area}[AXP] \cong \text{Area}[ANQ] \cong \text{Area}[BMS] \cong 2\text{Area}[MXP] \cong 2\text{Area}[MXQ] \cong 2\text{Area}[MXS]$

$\text{Area}[CXP] \cong \text{Area}[DNQ] \cong \text{Area}[CMS] \cong 2\text{Area}[NXP] \cong 2\text{Area}[NXQ] \cong 2\text{Area}[NXS]$

$\text{Area}[APQ] \cong \text{Area}[BPQ] \cong \text{Area}[APS] \cong \text{Area}[BPS] \cong \frac{1}{2}\text{Area}[AQS] \cong \frac{1}{2}\text{Area}[BQS]$

$\text{Area}[CPQ] \cong \text{Area}[DPQ] \cong \text{Area}[CPS] \cong \text{Area}[DPS] \cong \frac{1}{2}\text{Area}[CQS] \cong \frac{1}{2}\text{Area}[DQS]$

$\text{Area}[MPQ] \cong \text{Area}[NPQ] \cong \text{Area}[XPQ] \cong \text{Area}[MPS] \cong \text{Area}[NPS] \cong \text{Area}[XPS] \cong \frac{1}{2}\text{Area}[MQS] \cong \frac{1}{2}\text{Area}[NQS] \cong \frac{1}{2}\text{Area}[XQS]$